# Teaching mathematically gifted students in primary school

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The paper examines the changing conception of giftedness over the last four decades, and characteristics of mathematically gifted students. Ways of identifying mathematically gifted students in a study conducted with year 6 students are described. The Renzulli Enrichment Triad model for catering for gifted students is outlined. Case studies of a number of school and individual teacher's programs which cater for mathematically gifted students are described. A professional development module [CD ROM] designed for teacher inservice will be demonstrated.

Some schools and teachers have spent large amounts of time and energy helping students with learning difficulties and have paid scant attention to those students who demonstrate special talents in a whole range of areas. The exception to this is those students who show exceptional ability or potential on the sporting field. It is reasonable to conclude that we have treated these talented students inequitably when social justice and equity are driving forces in our education systems.

This paper looks at the construct of giftedness and how individual classroom teachers and schools identify gifted students and how they deal with these students in ways which maximise their gifts. It further focuses on gifted mathematics students and ways of identifying them. Classroom programs and whole school programs are discussed with reference, among other things, to the use of technology including the World Wide Web and email.

## Definitions of Giftedness

Braggett (1994) indicated that during this century, the notion of giftedness has moved from a unitary concept based on IQ measures to a multidimensional concept (Guilford, 1967) which included a range of abilities and incorporated both products and processes. Renzulli (1977) included Above-average Ability, Creativity and Task Commitment in his analysis of the components of giftedness, thereby indicating the importance of motivation as a determinant.

According to Gagne (1995), giftedness relates to five domains or ability areas - Intellectual, Creative, Socioemotional, Sensori-motor and Other. In his model, giftedness refers to ability while talent refers to performance or behaviour in specific areas. This distinction allows educators to shift the emphasis from 'the gifted' or 'the talented' or from 'being gifted or talented' to the development of gifted behaviours in students showing high potential for benefiting from special educational services (Renzulli & Reis, 1997).

Gardner's (1993) theory of multiple intelligences included linguistic intelligence, musical intelligence, logical mathematical intelligence, spatial intelligence, bodily kinaesthetic, intrapersonal and interpersonal intelligence. This lends support for the multidimensional nature of the concept. After analysing the various theoretical positions, Feldhusen (1989) concluded that giftedness consists of: superior general abilities; special focused aptitudes and/or talents; the acceptance that high level creative production is personally achievable; and the motivation to learn and achieve.

## Characteristics of Mathematically Gifted Students

Mathematics educators differ on what constitutes mathematical giftedness. Trafton (1981) claimed that the type of students who are labelled "gifted" and "talented" is not always clear, since the term is loosely applied to three classes of students: (a) those who learn standard content well and perform accurately, but who have difficulty when taught at a faster pace or deeper conceptual level; (b) those who learn more content and at a deeper level, who reason well, and who are capable of solving more complex problems than average students; and (c) those who are highly talented or precocious, performing at

the level of students several years older with little or no formal instruction. He seems to be suggesting that he would not consider the first two groups as mathematically gifted.

Greenes (1981) identified seven attributes that characterise mathematically gifted students, namely:(a) spontaneous formulation of problems; (b) flexibility in handling data; (c) data organisation ability; (d) mental agility or fluency of ideas; (e) originality of interpretation; (f) ability to transfer ideas; and (g) ability to generalise. These attributes could be applied to the last two groups mentioned by Trafton (1981). Johnson (1983) suggested that the characteristic that separates a gifted from a non gifted child in mathematics is the quality of the child's thinking; that is how the child reasons mathematically.

The most comprehensive list of characteristics of mathematically gifted students was provided by Lester and Schroeder (1983) in their summary of the work of Russian psychologist, V. A. Krutetskii which follows:

1. Perceive mathematical material in a problem both analytically and synthetically.
2. Generalise quickly and broadly both the content of a problem and the method of solution.
3. Curtail processes for solving similar problems with relatively little exposure.
4. Switch easily from one cognitive process to another even if it is qualitatively different.
5. Are independent of conventional solution techniques and rather readily reconstruct thought patterns.
6. Strive for elegant solutions (i.e., simple, economical solutions which are direct, general, and clear.
7. Reverse reasoning processes easily.
8. Thoroughly investigate aspects of difficult problems before directly attempting to solve them.
9. Remember generalised and curtailed structures associated with problems and their solutions. (This may help explain the sudden flashes of insight often experienced by these students.)
10. Tire both mentally and physically less readily during mathematical activity than during other kinds of lessons.
11. Develop abilities in stages. (Ability to generalise, appears first, curtailed reasoning, interest in elegance, and generalised, curtailed memory later.)
12. View the world mathematically; have a mathematical cast of mind.

It is evident from this discussion that mathematical giftedness has many dimensions which will be exhibited in varying degrees by students in our classrooms. Teachers are faced with the task of identifying those who are potentially gifted so that they can offer them learning experiences that will develop their gifts to the full. In the next section ways of making such identification are examined.

Procedures for the Identification of Mathematically Gifted Students

Since giftedness is widely regarded as a multifaceted construct, Landvogt (1991) recommends choosing a number of identification measures when seeking to identify mathematically gifted students. She suggests including mathematical problem-solving performance, teacher nomination, peer or self nomination, and parent nomination to name a few.

Adibnia (1996), in a study which sought to enhance the problem-solving performance of year 6 students, used two measures of problem-solving ability to identify the 'high ability' or 'gifted' students. The first was a test he devised for measuring problem-solving performance and for which he used an holistic scoring scheme, and the second was the Collis, Romberg and Doig (1992) Mathematical Problem Solving Profiles. Adibnia (1996) also had each classroom teacher complete a teacher-nomination checklist with rated students on a number of criteria related to mathematical giftedness and the pupils were required to fill in a self-rating checklist on their perceptions of

themselves and mathematical giftedness compared with their peers. He then was able to rank order the students on all four measures and combine these rankings to gain an overall ranking of the students. From this he selected those in the top 25% to the 'high ability' or 'mathematically gifted' category, those in the bottom 25% in the 'low ability' category and the middle 50% in to the average category.

## The Enrichment Triad Model

Renzulli and Reis (1997) have developed a School-wide Enrichment Model [SEM] for identifying and developing those students who exhibit gifted behaviours in many different areas. Their model is based on Renzulli's (1977) Enrichment Triad Model (Figure 1) which is sometimes linked with the Revolving Door Identification Model (Renzulli, Reis, & Smith, 1981) which allows for students to move into and out of Type III activities depending on their level of interest and the like. For Renzulli (1977), enrichment means "experiences or activities that are above and beyond the so called "regular curriculum" (p. 19).
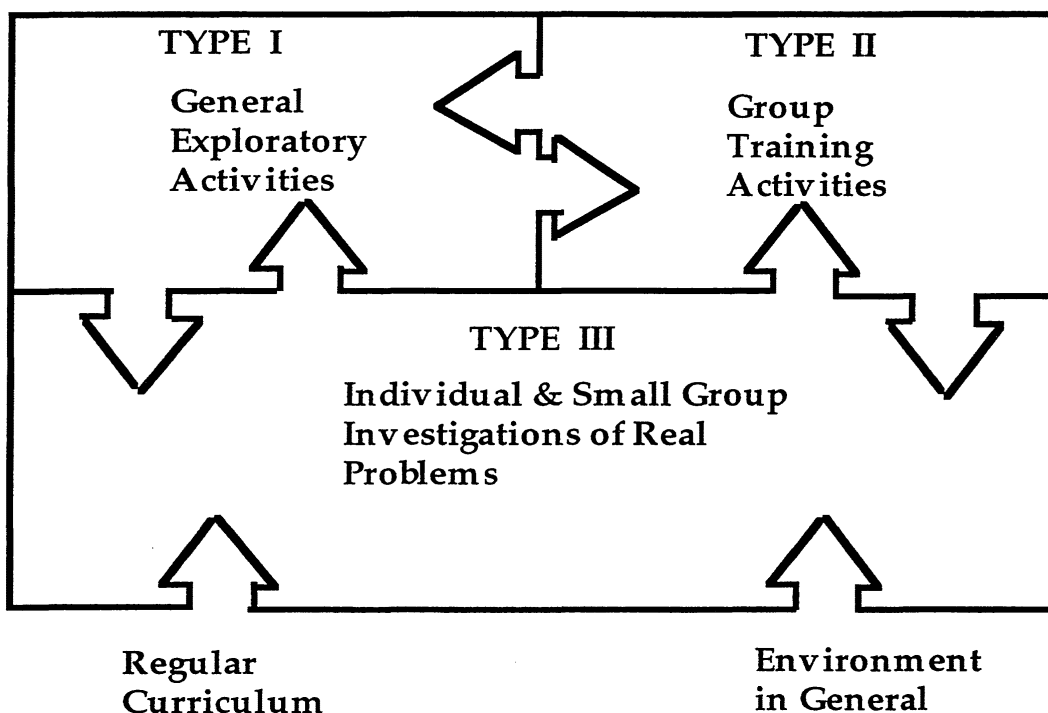


**TYPE I**

General
Exploratory
Activities

**TYPE II**

Group
Training
Activities

**TYPE III**

Individual & Small Group
Investigations of Real
Problems

Regular
Curriculum

Environment
in General

Figure 1. Enrichment Triad Model (Renzulli, 1977, p. 20)

*Type I Enrichment Activities*

These are generally available to and appropriate for all students, however, the major objectives for these activities are (i) to assist teachers and students in identifying those topics that some students may follow in depth, and (ii) to assist teachers in making decisions about the Type II enrichment activities that are required for particular students to progress further. Some examples of these Type I activities are:

1. Interest centres where student interest in a topic can be aroused and the student can explore the topic further outside the centre.

2. Listening to and discussing with resource people who are actively and creatively involved in their own professional or artistic field and who could be invited to discuss with potential gifted students their particular area of expertise or interest.

3. Visits to workplaces which involve more than just information gathering although for some students this might be the beginning of their interest. If high ability students can

ask questions and interact with people in their workplace, they can delve more deeply into a particular interest.

4. Students' interests sometimes are awakened through magazines on a particular topic or a video or TV program which covers the topic. With the CD versions of encyclopedia so readily available, some students' interests may be aroused as they explore topics on the computer.

5. There are many specific books on mathematics topics for children from primary level through to tertiary. History of mathematics and famous mathematicians are both interest areas for some students. Sometimes these are found in bookstores or libraries. Some of the journals for mathematics teachers have articles which may stimulate some students.

6. Investigations with mathematics equipment such a pantograph, a planimeter, or a clinometer may lead interested students to dig into the history of measurement and measurement devices.

7. Calculator explorations and computer software of varying types also have the potential to generate interest.

*Type II Enrichment- Group Training Activities*

These are designed to develop learners' abilities to deal more effectively with content which formed part of the Type I activities. Most Type II Enrichment involves training activities or process-oriented teaching activities which major on the development of thinking and feeling processes. Their focus is on developing specific skills involved in: (a) creative thinking, (b) critical thinking, (c) reflective thinking, (d) independent study, (e) divergent thinking, (f) inquiry training, (g) self-directed learning, (h) sensitivity training, (i) lateral thinking, (j) research, (k) inductive reasoning, (l) problem solving, and (m) awareness development. In fact, the higher levels of the Bloom, Hastings, and Madaus taxonomy of educational objectives (1971), namely, analysis, synthesis, and evaluation are often considered when looking at Type II Enrichment Activities.

While the focus in these types of enrichment activities are on developing processes, it is important to remember that this process development is a means to an end, rather than an end in itself. The experiences should arise out of the genuine interests that students exhibit in the Type I experiences and also prepare them for the Type III Enrichment which flows on from Type II activities.

*Type III Enrichment - Individual and Small Group Investigations*

These consist of activities in which learners' investigate real problems or topics. They use the skills developed in Type II activities to formulate a problem to be investigated and the methods to apply in dealing with its solution. To be an authentic Type III activity the students should be involved in investigation/research/production similar to a professional working in the same field. The outcome of such an activity should be the generating of a real product to be reported to a real audience.

Renzulli (1977) claims that the teacher's role in Type III Enrichment is more managerial as a resource person whose job is to help identify a student's interest and then to encourage the student to pursue this interest in an investigative way with a view to a creative/productive outcome. Once the student has refined the focus of his/her interest the teacher can then assist in the selection of a real audience to which the product of the investigation can be presented in whatever form.

Examples of School Programs Catering for Gifted Students

*Secondary School*

Two schools which have taken steps to cater for students with high ability some of whom may be classed as mathematically gifted have done so in different ways to allow for the different programs operating in their school. One is a country high school which saw the need to cater for a large range of ability in year 8 mathematics students. In order to accomplish this, it was decided to arrange clusters of two or three classes to be timetabled concurrently. Year 7 data on the students' performance on problem solving, computational skills, general interest areas and possible behaviour management problems were used to allocate students to the various class groupings. In this way, both the high

ability and the low ability and special needs students received learning experiences which were appropriate to their needs. A group of teachers who opted to take part in this program worked to cover the regular curriculum content but also developed appropriate resources which provided extension activities for the high ability students. Evaluation by the students, teachers and parents have been quite positive and there were plans to extend the scheme to other year 8 subjects and into years 9 and 10.

*Primary School*

The second school is a parochial school in a large city in tropical Queensland which has adopted a policy of flexible purposeful grouping as well as home classes across a number of grades. This is accompanied by workshops where gifted students are withdrawn for a whole day to cover specific activities in areas such as drama and writing. Students are placed into four mathematics levels at the beginning of the year based on testing of the previous years content as well as some off-level testing. There are two high ability groups, one or middle ability groups, and a lower ability group which is kept small [12-15] to allow the students who need the extra attention to receive it. Students' performance, effort and ability are constantly monitored and students can be moved to a higher or lower level in the light of these data. Students who feel they are not handling a particular level can also request to move to a lower level when this occurs. This is an important principle that the school is strongly committed to and it means that students do not feel that they are trapped in the same level for a whole year. Assessment is continuous and, because there are home class mathematics lessons as well as lessons in the flexible grouped arrangement, it is shared between the regular class teacher and the teacher in the particular ability level.

The focus of mathematics learning in the high ability group is on mathematical processes and thinking skills. Problem solving with universal applications and divergent thinking are developed through individual and small group investigations with materials from the Maths Task Centre Kit (Curriculum Corporation, 1993), the Mathematics Curriculum and Teaching Program [MCTP] (Lovitt & Clarke, 1988) and from materials used to prepare students for the Mathematics Olympiad and the various mathematics competitions such as Westpac and for activities relating to the annual Tournament of Minds competition. However, when high ability students are in their regular classes the teachers use curriculum compacting as a way to buy time for these students to use on extension activities.

Examples of Individual Classroom Programs for Gifted Students

*Lower Primary School:*

One example of how a teacher caters for gifted students within her classroom is taken from another school in the city mentioned earlier in tropical Queensland. The teacher has a year 1, 2, and 3 multi-age class which includes children ranging from gifted to intellectually impaired. Her formal teaching focuses on a particular concept with planning of activities to develop the concept based on the various levels of Bloom et. al. (1971) taxonomy of educational objectives and Gardner's (1993) theory of multiple intelligences. In this way she caters for the different ability levels within this class grouping.

Two other strategies used to individualise instruction are learning centres with a full range of activities based again on Bloom's taxonomy and projects whose topics are chosen voluntarily by the children. A problem-solving approach is the most common strategy for project work. Students who tested out of the current mathematics unit chose to identify the most fierce dinosaur and then identified height, mass, teeth, claws, and speed as key factors which they would need to research. As a conclusion to their project they wrote a report and presented their findings to the class.

The school also enters students in the Tournament of Minds competition and, in 1996, allowed younger students to be part of the teams with older students. In 1995 students in this grade won a prize for a big book they wrote as an entry in the Queensland Association of Mathematics Teachers competition.

*Upper Primary School:*

Joshua was in year 6 in 1996 at a small urban school which was located in a low socio-economic area of the same tropical city mentioned above. He was found to be gifted in literacy, numeracy and computer technology and in his second year at school learned Japanese with year 6 and 7 students and in his year 6 class was learning Japanese at a level way above his grade level. He was the resident expert on computers and was called upon to assist teachers with their software and hardware difficulties. He used multimedia tools to produce his reports which include audio and video segments which he developed and programmed.

In mathematics Joshua spent a great deal of his time pursuing individual projects and investigations like those in the Maths Task Centre Kit (Curriculum Corporation, 1993), but he was also always involved in group investigations in mathematics within his own class where he was a popular leader.

Resources for Assisting Teachers of Mathematically Gifted Students

As mentioned earlier, there are a number of commercially available kits such as the MCTP (Lovitt & Clarke, 1988) materials and the Maths Task Centre Kit which are excellent for extending mathematically gifted students. These can be used with all students at some point in their mathematics learning but also serve the purpose well as extension and enrichment for gifted students.

Mathematics competitions such as the Westpac Maths competition, the Queensland Association of Mathematics Teachers competition and the Mathematics Olympiad can serve as a valuable avenue for catering for gifted students. Locally run mathematics camps can often be an opportunity for Type I activities where students' interests in environmental or outdoor mathematics may begin.

Another medium of growing importance in catering for gifted students is the Internet. There are many locations on the WWW devoted to mathematics for school students and these sites are generally excellent sources for teachers who are seeking new materials and ideas for broadening their program for gifted students. A brief listing of some URL's found to be of value for mathematically gifted students follows:

*Maths Lessons*
  http://forum.swarthmore.edu/~steve/steve/mathlessons.html
  http://www.eskimo.com/~user/zcurric.html
  http://www.mcrel.org/connect/lesson.html#math
  http://carson.enc.org:80/lesmath.htm
*Maths Association of Victoria HOME PAGE*
  http://www.srl.rmit.edu.au/mav/
*QLD ASSN of Gifted & Talented*
  http://www.maxlink.com.au/qagtc/welcome.htm
*Internet Hot Spots*
  http://www.srl.rmit.edu.au/mav/intsites/index.html
*OZ- Gifted*
  http://owl.qut.edu.au/common-cgi-bin/forum/OZ-GIFTED/
*Fibonacci Numbers and Nature*
  http://www.ee.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html
*Activities for classroom on pi*
  http://www.ncsa.uiuc.edu:80/edu/RSE/RSEorange/Piactivities.html
*Problems set out in grade levels  solutions posted each week*
  http://www.hmco.com/school/math/brain/
  http://www.pacificnet.net/~mandel/Math.html
*Classroom, the Twelve Days of Christmas*
  http://www.feist.com/~lshiney/lesson/math.html
*Great photograph activities all maths related*
  http://acorn.educ.nottingham.ac.uk/Maths/photomath/

*Union Elementary School - collection of problems each week*
    http://plainfield.bypass.com/~union/problems.html
*Maths competitions and clubs*
    http://cq-pan.cqu.edu.au/schools/smad/mathcomp.html
*Wonderful World of Mathematics*
    http://plainsfield.bypass.com/~union/world.html
*Mega Mathematics*
    http://www.c3.lanl.gov/mega-math/menu.html
*Houghton Mifflin Education Place main index.*
    http://www.hmco.com/hmco/school/main.html
*Uni of Connecticut*
    http://www.ucc.uconn.edu:80/~wwwgt/
*Renzulli Centre*
    http://www.ucc.uconn.edu:80/~wwwgt/nrcgt.html
*Wyoming Future Problem Solving Program*
    http://www.uwyo.edu/a&s/nasc/nasc2.htm

Some teachers have taken advantage of the WWW to introduce their potentially gifted students to Type I activities and then the students have been able to carry out their own investigations using the WWW. It seems that the opportunities are almost unlimited for us as teachers if we can explore this resource for and with our students.

Conclusion

In this paper an attempt has been made to cover a number of areas related to teaching mathematically gifted students. One of the main challenges for classroom teachers is to find time to collect resources which will extend their gifted students to their fullest potential. The author has produced a professional development package on *Teaching Mathematically Gifted Students* which is delivered on CD ROM to people in their own home or workplace.

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